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ACTIVE CONTROL OF CANTILEVERED PIPES CONVEYING FLUID WITH CONSTRAINTS ON INPUT ENERGY

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This paper deals with the stabilization of flutter in cantilevered pipes conveying fluid with constraints on input energy for control. The controller in this study is designed by using an algorithm which iteratively tunes the weighting matrix of the quadratic performance index in the LQG problem. This controller design method takes the performance of the actuator into consideration. The theoretical and experimental results show that the effectiveness of the controller in stabilizing flutter in cantilevered pipes conveying fluid varies according to the input variance constraints, i.e., the difference in the dynamic range of an actuator. Furthermore, the critical flow velocity at instability of the pipe can be estimated by the method stated in this paper, where the system loses its stability because of the restriction of control force produced by an actuator.

1. INTRODUCTION

THE STUDY OF THE BEHAVIOUR of cantilevered pipes conveying fluid is not only important in the engineering field (oil pipelines, heat exchanger tubes, etc.), but also academically interesting as a nonconservative problem of elastic stability. Several researchers have investigated this problem from the viewpoint of dynamic stability (Gregory & Païdoussis 1966 a, b; Païdoussis & Issid 1974; Sugiyama *et al.* 1985, 1988). However, few studies have been reported on the stabilization of cantilevered pipes conveying fluid except the authors' previous works. In these studies, first of all, the feasibility of applying the optimal control theory for stabilizing a cantilevered pipe conveying fluid was investigated theoretically (Doki & Tani 1986, 1988). Next, the effects of the mass of the control device on the dynamic stability and active control of a pipe and the most suitable location of the control device were studied theoretically (Doki & Aso 1989). The required force for the active control of the pipe was produced by two methods: one was the tendon-control method for a moment actuator and the other was the thruster-control method for a transverse force actuator. It

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was shown experimentally by Doki *et al.* (1995) that the simplified active control method using an analog PID controller, in which no CPU was included in the control systems, was effective for stabilization of the cantilevered pipe. Furthermore, they clarified that the H^{∞} controller was robust against the change in properties of the pipe system caused by increasing flow velocity, in comparison with a PID controller (Doki *et al.* 1996). Recently, a study on the active control of chaotic vibration of a pipe was reported by Yau *et al.* (1995).

Although passive vibration-control methods have usually been used for vibration suppression in the engineering field, their effectiveness decreases considerably when the design parameters and/or characteristics of the system vary. Instead of these methods, an active vibration-control method which uses external energy has been introduced to many engineering applications in order to suppress the vibration. However, an actuator cannot produce an unlimited control force in the active control of structures. Therefore, it becomes important to investigate theoretically and experimentally the effect of the performance of an actuator on the dynamic stability and active control of cantilevered pipes conveying fluid.

This paper deals with the stabilization of flutter in cantilevered pipes conveying fluid with a constraint on input energy for control, while minimizing the variance of the pipe deflection. We take the input energy for control as an index of the performance of the actuator and try to design a controller which satisfies the constraint condition on the variance of the closed-loop control effort. This controller design method takes the performance of the actuator into consideration, and yields an algorithm which iteratively tunes the weighting matrix of the quadratic performance index in the LQG problem (Hsieh *et al.* 1989; Zhu & Skelton 1991; Zhu *et al.* 1997).

The control object which consists of a pipe system and an actuator is modelled mathematically using the Galerkin method. The experimental set-up consists of the pipe system, a laser sensor, an A/D converter, a personal computer, a D/A converter, and a DC servomotor which produces a transverse force for active control of the pipe. It will be shown from the theoretical and experimental results that the effectiveness of the controller in stabilizing the flutter phenomenon in cantilevered pipes conveying fluid varies according to the input variance constraints, i.e., the difference in the performance of an actuator. The input variance constraints stated in this paper can be used as a guide for the selection of an actuator when the active control system is designed. Furthermore, it will be shown that the critical flow velocity at instability of the pipe can be estimated by the method given in this paper, where the system loses its stability because of the restriction of the control force produced by the actuator.

2. ANALYTICAL MODEL OF CANTILEVERED PIPES CONVEYING FLUID

The thruster-controlled pipe system under consideration is shown in Figure 1. It consists of a uniform cantilevered pipe of length L, flexural rigidity EI, mass per unit length m_b, and the control system. Furthermore, an incompressible fluid of mass per unit length m_f flows with a constant velocity V inside the pipe. The pipe is supported horizontally by N_s strings of length l, and for modelling purposes the pipe is assumed to move only in the horizontal plane. Each tension $T_i = C_i(m_f + m_b)gl/N_s$ ($C_i > 0$, $i = 1, \dots, N_s$) is obtained by using Clapeyron's Three Moment Theorem for a continuous beam. In order to control the response of the cantilevered pipe, a target of a sensor and a thruster with a spring constant K, whose lumped mass is negligible, are attached to the pipe at $x = L_s$ and L_a, respectively.

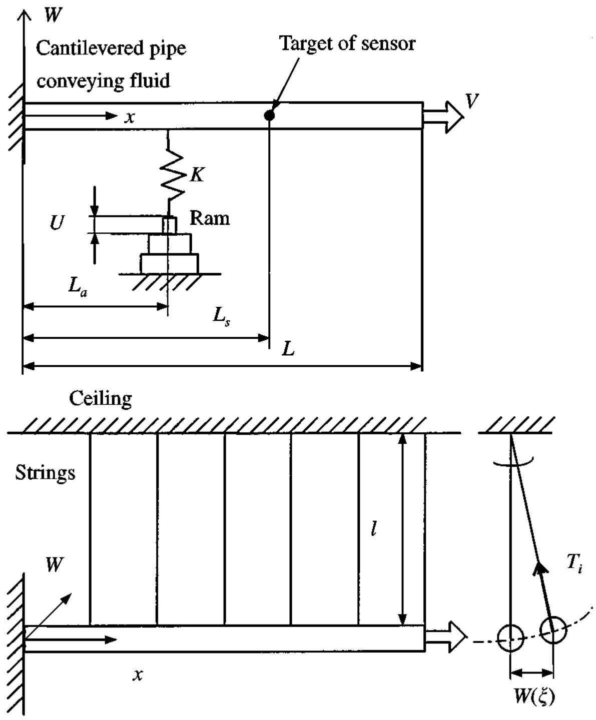

Figure 1. Definition of the analytical model.

The control displacement $U(t)$ is determined by a control law, which will be stated in the next section, based on the response of the pipe. The thruster creates a transverse force by the action of the ram at $x = L_a$. It is assumed that the pipe is made of a material with Kelvin–Voigt type viscoelasticity, and that E^* is the coefficient of internal dissipation. With $W(x, t)$ denoting the deflection of the cantilevered pipe and taking the effects of horizontal force based on the tension T_i of supporting strings of the pipe into consideration, the governing differential equation of motion of the pipe and the boundary conditions are given in the nondimensional form as follows (Païdoussis & Issid 1974; Doki *et al.* 1996):

$$L(w) = w_{,\xi\xi\xi\xi} + \mu w_{,\xi\xi\xi\xi\tau} + 2\sqrt{\beta} v w_{,\xi\tau} + v^2 w_{,\xi\xi}$$

$$+ w_{,\tau\tau} + k(w - u)\delta(\xi - \xi_a) + \tau_s \sum_{i=1}^{N_s} C_i w(\xi)\delta(\xi - \xi_i) = 0, \tag{1}$$

$$w(0, \tau) = w_{,\xi}(0, \tau) = w_{,\xi\xi}(1, \tau) + \mu w_{,\xi\xi\tau}(1, \tau)$$

$$= w_{,\xi\xi\xi}(1, \tau) + \mu w_{,\xi\xi\xi\tau}(1, \tau) = 0, \tag{2}$$

where the subscript after a comma denotes the partial differentiation with respect to the corresponding independent variable (ξ or τ) and $\delta(\xi)$ is the Dirac delta function. The nondimensional quantities in these equations are related to the physical ones through the following relations:

$$w = \frac{W}{L}, \quad \xi = \frac{x}{L}, \quad \xi_a = \frac{L_a}{L}, \quad \xi_s = \frac{L_s}{L},$$

$$\beta = \frac{m_f}{m_f + m_b}, \quad u = \frac{U}{L}, \quad v = VL\sqrt{\frac{m_f}{EI}},$$

$$\mu = \frac{E^*}{E}\frac{1}{L^2}\sqrt{\frac{EI}{m_f + m_b}}, \quad k = \frac{KL^3}{EI}, \tag{3}$$

$$\tau = \frac{t}{L^2}\sqrt{\frac{EI}{m_f + m_b}}, \quad \tau_s = \frac{(m_f + m_b)gL^4}{lN_sEI},$$

in which t is the time. Considering boundary conditions (2), the deflection $w(\xi, \tau)$ can be represented by the following equations:

$$w(\xi, \tau) = \sum_{m=1}^{N} a_m(\tau)\phi_m(\xi), \tag{4}$$

$$\phi_m(\xi) = (\cosh\alpha_m\xi - \cos\alpha_m\xi) - \sigma_m(\sinh\alpha_m\xi - \sin\alpha_m\xi),$$

$$\sigma_m = \frac{\sinh\alpha_m - \sin\alpha_m}{\cosh\alpha_m + \cos\alpha_m}, \tag{5}$$

where $a_m(\tau)$ is an unknown time function, $\phi_m(\xi)$ is a normalized eigenfunction of the cantilever and α_m is a solution of the following frequency equation:

$$1 + \cosh\alpha_m\cos\alpha_m = 0. \tag{6}$$

Substituting equation (4) into equation (1) and applying the Galerkin procedure, i.e., multiplying the resulting equation by ϕ_r and integrating it from 0 to 1, the following equations are obtained:

$$\sum_{m=1}^{N}(D_{rm}\ddot{a}_m + E_{rm}\dot{a}_m + F_{rm}a_m) = G_r u(\tau), \quad (r = 1, 2, \ldots, N), \tag{7}$$

$$D_{rm} = \delta_{rm}, \quad E_{rm} = 2\sqrt{\beta}vb_{rm} + \mu\alpha_m^4\delta_{rm},$$

$$F_{rm} = \alpha_m^4\delta_{rm} + v^2c_{rm} + k\phi_r(\xi_a)\phi_m(\xi_a) + \tau_s\sum_{i=1}^{N_s}C_i\phi_r(\xi_i)\phi_m(\xi_i),$$

$$G_r = k\phi_r(\xi_a),$$

$$b_{rm} = \frac{4}{(\alpha_r/\alpha_m)^2 + (-1)^{r+m}}, \tag{8}$$

$$c_{rm} = \begin{cases} \dfrac{4(\alpha_m\sigma_m - \alpha_r\sigma_r)}{(-1)^{r+m} - (\alpha_r/\alpha_m)^2} & (r \neq \mathrm{m}) \\ \alpha_r\sigma_r(2 - \alpha_r\sigma_r) & (r = m), \end{cases}$$

where the overdot denotes differentiation with respect to the nondimensional time τ, and δ_{rm} is the Kronecker delta. Equation (7) is rewritten in state-space form as follows:

$$\dot{\mathbf{x}}(\tau) = \mathbf{A}\mathbf{x}(\tau) + \mathbf{B}u(\tau),$$
$$\mathbf{x}(\tau) = [a_1, a_2, \ldots, a_N, \dot{a}_1, \dot{a}_2, \ldots, \dot{a}_N]^{\mathrm{T}}, \tag{9}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{D}^{-1}\mathbf{F} & -\mathbf{D}^{-1}\mathbf{E} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{D}^{-1}\mathbf{G} \end{bmatrix},$$
$$\mathbf{D} = [D_{rm}], \quad \mathbf{E} = [E_{rm}], \quad \mathbf{F} = [F_{rm}], \quad \mathbf{G} = [G_r]. \tag{10}$$

In the above equation, $[\,]^{\mathrm{T}}$ means the transpose of the matrix. The deflection of the pipe at $\xi = \xi_s$ is chosen as the output of the system

$$w(\xi_s, \tau) = \mathbf{M}\mathbf{x}(\tau),$$
$$\mathbf{M} = [\phi_1(\xi_s), \quad \phi_2(\xi_s), \ldots, \phi_N(\xi_s), \quad 0, \ldots, 0]. \tag{11}$$

Figure 2 shows a comparison of Bode diagrams from u to $w(\xi_s, \tau)$ of the pipe system between the analytical model, which is analysed by using the five-mode approximation ($N = 5$), and the experimental data for flow velocity of water $V = 0$ m/s [Figure 2(a)] and $V = 6{\cdot}74$ m/s [Figure 2(b)]. It is found that the analytical model for the cantilevered pipe conveying fluid in this study is appropriate because the analytical results agree well with the experimental values.

3. CONTROLLER DESIGN METHOD

3.1. Input Variance Constraint (IVC) Controller Design

In this study, we formulate the controller design method, taking the performance of the actuator into consideration, as a controller design problem which satisfies the following specifications.

Problem: *For a system with white noise in the plant and measurement, design a controller which minimizes the variance of the closed-loop response subject to the constraints on input variance.*

This problem is described mathematically as the following IVC control problem (Skelton 1988; Hsieh *et al.* 1989). Using equations (9)–(11), consider the following system:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{L}_v\mathbf{v},$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}, \tag{12}$$
$$z = \mathbf{M}\mathbf{x} + \mathbf{L}_w\mathbf{w},$$

where $\mathbf{v} \in \mathbf{R}^{\bar{l}}$ and $\mathbf{w} \in \mathbf{R}^{\bar{m}}$ are zero-mean white noise vectors with intensity $\mathbf{V} > 0$ and $\mathbf{W} > 0$, respectively. Furthermore, we assume that there is no correlation between $\mathbf{v}$ and $\mathbf{w}$, $\mathbf{y} \in \mathbf{R}^{\bar{n}}$ and $z \in \mathbf{R}^1$ are the controlled output vector and the measurement, respectively. In this paper, the matrix $\mathbf{C}$ is determined as

$$\mathbf{C} = [\phi_1(1), \ldots, \phi_N(1), \quad 0, \ldots, 0]. \tag{13}$$

This means that the controlled output vector $\mathbf{y}$ is the displacement at the free end of the pipe. Since all modes of vibration are observable at the free end of the cantilevered beam, the problem on observability does not arise. $\mathbf{L}_v$ and $\mathbf{L}_w$ are the real $(2N \times \bar{l})$- and $(1 \times \bar{m})$-dimensional matrices. For the system in equation (12), consider the design problem

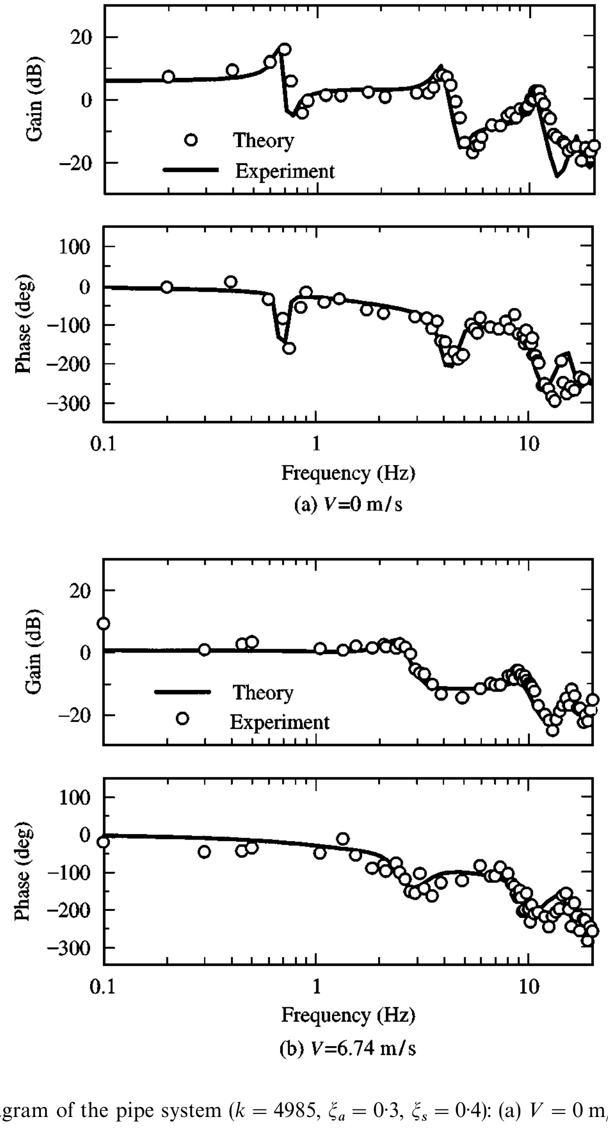

Figure 2. Bode diagram of the pipe system ($k = 4985$, $\xi_a = 0{\cdot}3$, $\xi_s = 0{\cdot}4$): (a) $V = 0$ m/s; (b) $V = 6{\cdot}74$ m/s.

to minimize

$$J = E_\infty\{y^{\mathrm{T}}\mathbf{Qy}\}, \tag{14}$$

$\mathbf{Q}$: $\bar{n} \times \bar{n}$ positive symmetric matrix

subject to the inequality constraints

$$E_\infty\{u^2\} \le \lambda \tag{15}$$

where E_∞ is the expectation operator and λ (>0) means an upper bound of the variance of control input u specified by a control system designer.

3.2. Computational Algorithm for IVC Control Problem

The IVC problem can be solved by using the following design algorithm (Hsieh *et al.* 1989; Zhu & Skelton 1991; Zhu *et al.* 1997) which iteratively tunes the weighting matrix of the quadratic performance index in the LQG problem. We identify the following steps.

Step 1. Design beforehand the Kalman filter for system (12) and assume the Kalman filter gain $\mathbf{H}$. Specify $\lambda(>0)$ in equation (15). Set $r(0)>0$ and the iteration number j as 1.

Step 2. Let $r(j)>0$. Obtain the optimal regulator gain $\mathbf{G}(j) = -\mathbf{B}^{\mathrm{T}}\mathbf{P}(j)/r(j)$ which minimizes the following quadratic cost function:

$$\tilde{J} = E_{\infty}\{\mathbf{x}^{\mathrm{T}}\mathbf{C}^{\mathrm{T}}\mathbf{Q}\mathbf{C}\mathbf{x} + r(j)u^2\}, \tag{16}$$

where $\mathbf{P}(j)$ is the solution of the following algebraic Riccati equation:

$$\mathbf{A}^{\mathrm{T}}\mathbf{P}(j) + \mathbf{P}(j)\mathbf{A} - \frac{\mathbf{P}(j)\mathbf{B}\mathbf{B}^{\mathrm{T}}\mathbf{P}(j)}{r(j)} + \mathbf{C}^{\mathrm{T}}\mathbf{Q}\mathbf{C} = \mathbf{0}. \tag{17}$$

Step 3. The state-space form of LQG controller is given as follows:

$$\begin{aligned}\dot{\mathbf{x}}_c &= \mathbf{A}_c\mathbf{x}_c + \mathbf{B}_c z,\\ u &= \mathbf{C}_c\mathbf{x}_c,\\ \mathbf{A}_c(j) &= \mathbf{A} + \mathbf{B}\mathbf{G}(j) + \mathbf{H}\mathbf{M}, \mathbf{B}_c = \mathbf{H}, \quad \mathbf{C}_c = \mathbf{G}(j).\end{aligned} \tag{18}$$

Define matrices as follows:

$$\begin{aligned}\mathbf{A}_0 &= \begin{bmatrix}\mathbf{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}, \quad \mathbf{B}_0 = \begin{bmatrix}\mathbf{B} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix},\\ \mathbf{C}_0 &= [\mathbf{C}\ \mathbf{0}], \quad \mathbf{D}_0 = \begin{bmatrix}\mathbf{L}_v\\ \mathbf{0}\end{bmatrix},\\ \mathbf{R}_0 &= \begin{bmatrix}1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}, \quad \mathbf{M}_0 = \begin{bmatrix}\mathbf{M} & \mathbf{0}\\ \mathbf{0} & \mathbf{I}\end{bmatrix},\\ \mathbf{G}_0(j) &= \begin{bmatrix}0 & \mathbf{C}_c(j)\\ \mathbf{B}_c & \mathbf{A}_c(j)\end{bmatrix}.\end{aligned} \tag{19}$$

Obtain the solution $\mathbf{X}(j)$ of the following Lyapunov equation:

$$(\mathbf{A}_0 + \mathbf{B}_0\mathbf{G}_0(j)\mathbf{M}_0)\mathbf{X}(j) + \mathbf{X}(j)(\mathbf{A}_0 + \mathbf{B}_0\mathbf{G}_0(j)\mathbf{M}_0)^{\mathrm{T}} + \mathbf{D}_0\mathbf{V}\mathbf{D}_0^{\mathrm{T}} = \mathbf{0}. \tag{20}$$

Using $\mathbf{X}(j)$, $E_{\infty}\{u^2\}$ is given as follows:

$$E_{\infty}\{u^2\} = \mathbf{R}_0\mathbf{G}_0(j)\mathbf{M}_0\mathbf{X}(j)\mathbf{M}_0^{\mathrm{T}}\mathbf{G}_0(j)^{\mathrm{T}} \tag{21}$$

Step 4. If

$$|\mathbf{R}_0[\mathbf{G}_0(j)\mathbf{M}_0\mathbf{X}(j)\mathbf{M}_0^{\mathrm{T}}\mathbf{G}_0(j)^{\mathrm{T}} - \mathbf{G}_0(j-1)\mathbf{M}_0\mathbf{X}(j-1)\mathbf{M}_0^{\mathrm{T}}\mathbf{G}_0(j-1)^{\mathrm{T}}]| < \varepsilon, \tag{22}$$

is satisfied, then stop. Otherwise, update $r(j)$ with

$$r(j+1) = r(j)\left[\frac{\mathbf{R}_0\mathbf{G}_0(j)\mathbf{M}_0\mathbf{X}(j)\mathbf{M}_0^{\mathrm{T}}\mathbf{G}_0(j)^{\mathrm{T}}}{\lambda}\right]^n, \tag{23}$$

by replacing j with $j+1$ and go to Step 2.

4. EXPERIMENT

4.1. Experimental Set-up

The experimental set-up and physical properties of the pipe are shown in Figure 3 and Table 1, respectively. The experiments are conducted with silicone rubber pipes conveying water. The pipe is supported horizontally at intervals of 0.1 m by six thin strings with length $l = 2{\cdot}2$ m hung from the ceiling because of its flexibility. This set-up consists of the pipe system, a laser sensor, an A/D converter, a personal computer which implements the discretized LQG controller discussed in the previous section, a D/A converter, and a DC servomotor which produces a transverse force for active control of the pipe. A target is attached at 0·24 m ($\xi_s = 0{\cdot}4$) from the fixed end of the pipe, so that a laser sensor can detect the deflection of the pipe. A control force acts on the pipe at 0·18 m ($\xi_a = 0{\cdot}3$) from the fixed

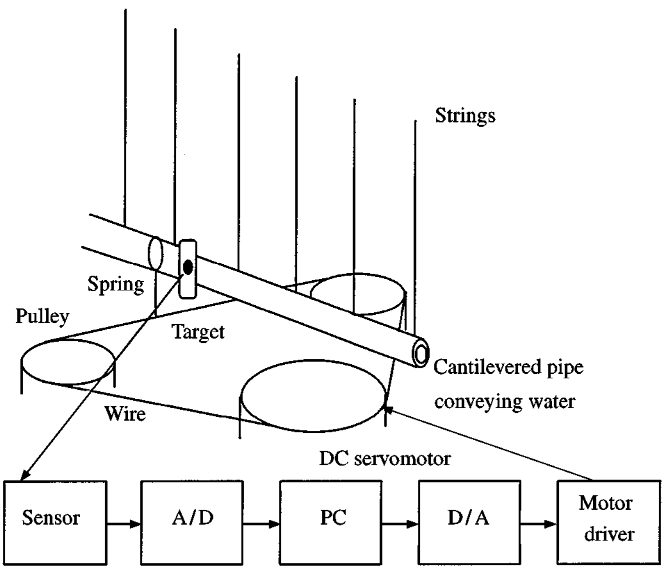

Figure 3. Experimental set-up.

Table I
Physical properties of pipe ($V = 0$ m/s)

Parameter	Value
Inner diameter, d (mm)	4·38
Outer diameter, D (mm)	12·23
Length, L (m)	0·602
Mass per unit length of pipe, m_b (kg/m)	0·122
Mass per unit length of fluid, m_f (kg/m)	$1{\cdot}52 \times 10^{-2}$
Young's modulus, E (Pa)	$6{\cdot}06 \times 10^{6}$
Natural frequency (1st mode), f_1 (Hz)	0·50
Natural frequency (2nd mode), f_2 (Hz)	2·25
Logarithmic decrement (1st mode), δ_1	0·15
Logarithmic decrement (2nd mode), δ_2	0·31

end of the pipe by using a DC servomotor, pulleys, a wire, and a spring with a nondimensional spring constant $k = 4985$ attached between the wire and the pipe.

4.2. Experimental Results

In the previous section, the inequality constraint on the variance of the control effort was defined as an index of performance of the actuator. In order to examine the effect of the constraint on the active control of cantilevered pipes conveying fluid, the following experiments were conducted. In these experiments, the pipe became unstable in the second mode by flutter in all cases. The controllers were designed for various values of λ by the method stated in the previous section. The convergence process of controller design is shown in Figure 4 for $\lambda = 4{\cdot}0 \times 10^6$ as an example.

A comparison between the controlled and uncontrolled responses of the pipe at $\xi_s = 0{\cdot}4$ and the variation of the control input u are shown in Figure 5(a–c) for $\lambda = 1{\cdot}0 \times 10^6$ and $6{\cdot}0 \times 10^6$ for a flow velocity $V = 7{\cdot}13$ m/s when the initial disturbance acts on the free end of the pipe. From this figure, it is observed that the pipe is unstable by flutter in the uncontrolled system, while the closed-loop systems for both values of λ suppress the disturbance and prevent the growth of flutter. Furthermore, the settling time of the response and the maximum value of the control input are about 3 s and 0·5 mm for $\lambda = 1{\cdot}0 \times 10^6$, and

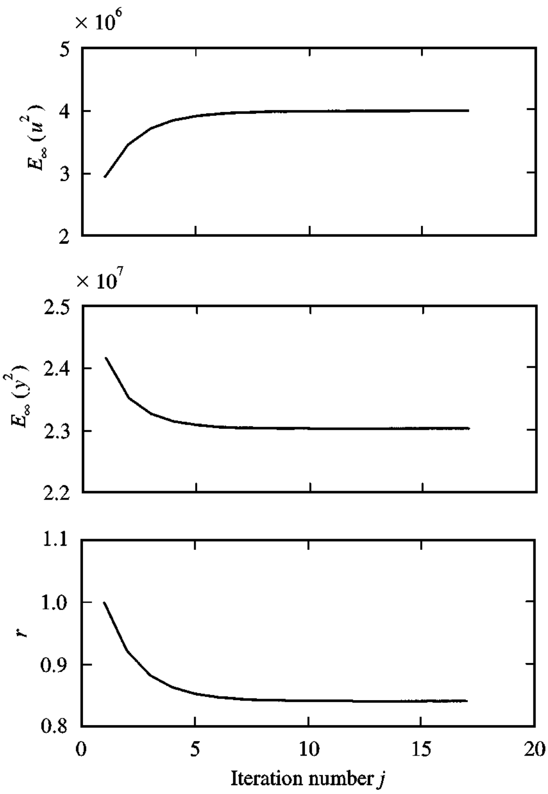

Figure 4. Convergence process of controller design ($\lambda = 4{\cdot}0 \times 10^6$).

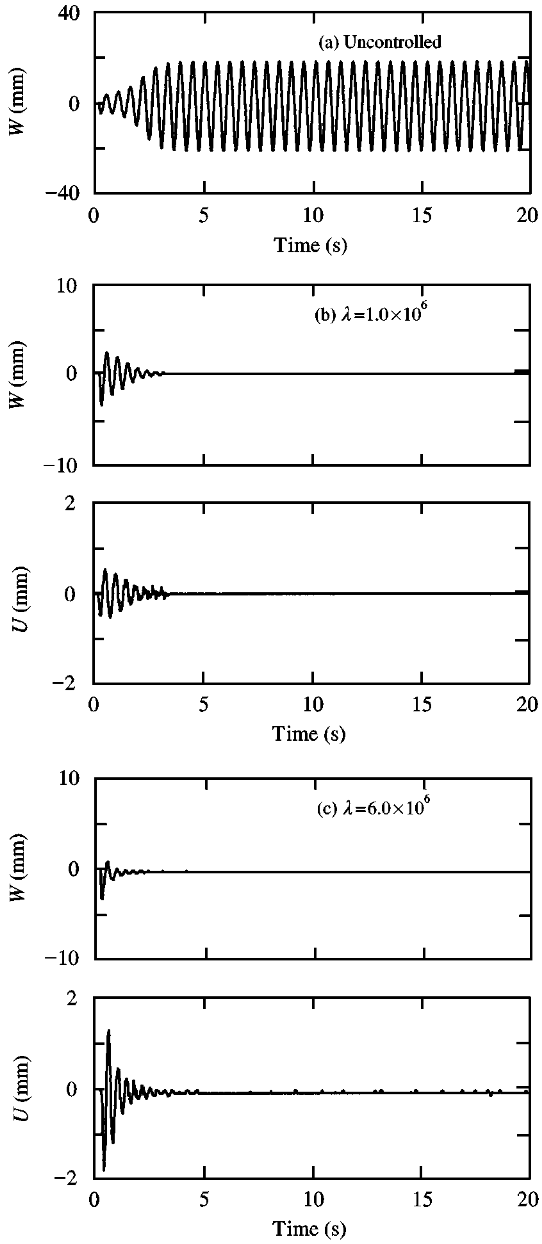

Figure 5. Response to initial disturbance ($k = 4985$, $\xi_a = 0{\cdot}3$, $\xi_s = 0{\cdot}4$, $V = 7{\cdot}13$ m/s).

about 1·5 s and 1·8 mm for $\lambda = 6{\cdot}0 \times 10^6$, respectively, because the latter controller is allowed to use more energy for control than the former. The stabilization effect of the controllers designed for $\lambda = 1{\cdot}0 \times 10^6$ and $6{\cdot}0 \times 10^6$ on flutter for a flow velocity $V = 6{\cdot}80$ m/s is shown in Figure 6. Figure 6(a) shows the uncontrolled flutter response. It is found that the flutter is completely controlled by switching on the controller at the

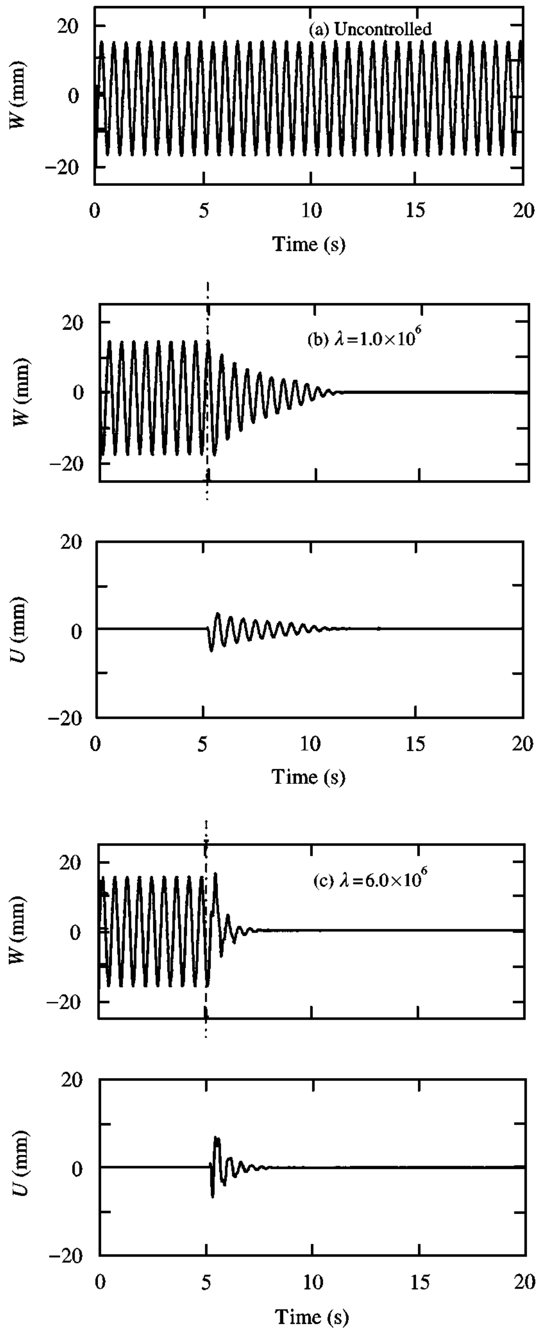

Figure 6. Flutter response (k = 4985, $\xi_a = 0{\cdot}3$, $\xi_s = 0{\cdot}4$, $V = 6{\cdot}80$ m/s).

chain-dotted line in Figure 6(a–c). The settling time of the response is about 3 s for $\lambda = 6{\cdot}0 \times 10^6$ and 43% of that for $\lambda = 1{\cdot}0 \times 10^6$. Figure 7 shows the response of the pipe to the flutter phenomenon for a flow velocity $V = 7{\cdot}13$ m/s. Although the pipe is stabilized by

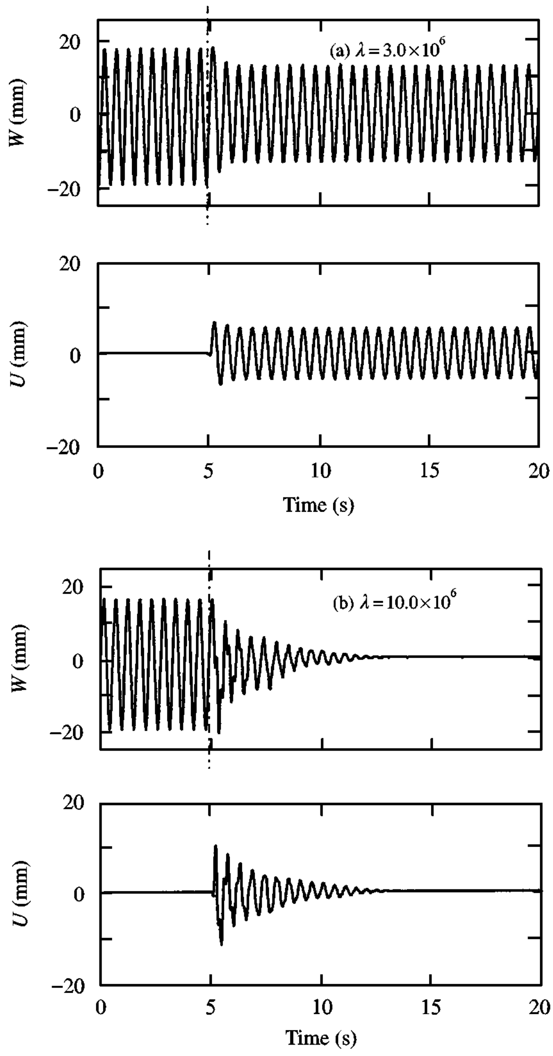

Figure 7. Flutter response ($k = 4985$, $\xi_a = 0{\cdot}3$, $\xi_s = 0{\cdot}4$, $V = 7{\cdot}13$ m/s).

the controller for $\lambda = 10{\cdot}0 \times 10^6$, the controller for $\lambda = 3{\cdot}0 \times 10^6$ cannot stabilize the pipe because the input energy for control is too small. However, the amplitude of the controlled response in this case is smaller than that of the uncontrolled one.

From these experiments, the controller described in this paper is found to be effective in stabilizing the flutter of a cantilevered pipe conveying fluid; its effectiveness varies according to the performance of the actuator.

Next, we examined theoretically and experimentally how the controller designed in this study increases the critical flow velocity when the system loses its stability, because of the restriction of control force produced by an actuator. If the control force is not sufficient because of deficiency in the performance of an actuator, the flutter is not controlled and the pipe becomes unstable at a new critical flow velocity. The theoretical critical flow velocity is

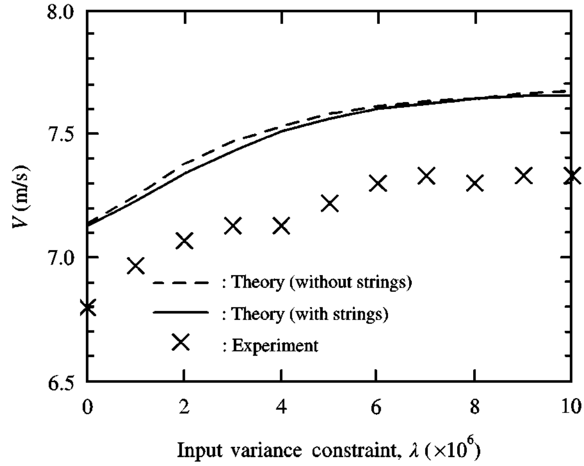

Figure 8. Critical flow velocity. The solid line gives theoretical critical flow velocity with supporting strings. The critical flow velocity without supporting strings is shown as the broken line.

computed by the following procedure in both cases: when the pipe is supported by six strings, and when no strings exist to support it.

Step 1. Design a controller for an energy constraint [λ in equation (15)] and a flow velocity V by the algorithm of Section 3.

Step 2. If the controller is designed successfully [the factor $r(j)$ is converged in the design algorithm] in Step 1 under this λ, V is replaced by V_{new} which is satisfied $V_{\text{new}} > V$ and go to Step 1.

Step 3. If the design of the controller is not successful [convergence of the factor $r(j)$ is not achieved], replace V by $V_{\text{new}} < V$ and to Step 1.

Using this procedure, the maximum flow velocity for which the stabilizing controller can be designed under some energy constraint (in the form of λ) is searched using a bisection-like method. Figure 8 shows the critical flow velocity for various values of λ. It is found that the critical flow velocity increases with an increase of λ, and theoretical predictions agree qualitatively well with experimental values.

5. CONCLUSION

The dynamic stability and active control of cantilevered pipes conveying fluid with constraints on input energy for control have been studied theoretically and experimentally. The controller in this study was designed by taking the variance constraints of the actuator into consideration. The effects of the constraints on the stabilization of the pipe conveying fluid have been considered. The main results are summarized as follows.

(i) The controller designed in this study is effective in stabilizing the cantilevered pipe conveying fluid.
(ii) The effectiveness of the controller in stabilizing the flutter phenomenon of cantilevered pipes conveying fluid varies according to the input variance constraints, i.e., the difference in the performance of an actuator. Therefore, the constraint condition on the

variance of the control effort stated in this study can be used as a guide for the selection of an actuator in the design of an active control system for cantilevered pipes conveying fluid.

(iii) The critical flow velocity of the pipe can be estimated by using the method given in this study, where the system loses its stability because of restriction of the control force produced by an actuator.

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